Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | | | | | |

Candidate Number | | | | |

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics
International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. The curve C has equation

$$y = (3x - 2)^6$$

(a) Find $\frac{dy}{dx}$

(2)

Given that the point $P\left(\frac{1}{3}, 1\right)$ lies on C,

(b) find the equation of the normal to C at P. Write your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

1. a) $y = (3x-2)^6$

$$\boxed{\text{CHAIN RULE}: \quad \frac{dy}{dx} = \frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{dx}}$$

$u = 3x - 2 \qquad \frac{du}{dx} = 3$

$y = u^6 \qquad \frac{dy}{du} = 6u^5$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 6u^5 \times 3 = 18(3x-2)^5$

b) $P = \left(\frac{1}{3}, 1\right)$

gradient of tangent at $P = \frac{dy}{dx} = 18\left(3\left(\frac{1}{3}\right) - 2\right)^5$

$= -18$

$\text{gradient}_{\text{normal}} = \frac{-1}{\text{grad. tangent}} \qquad \therefore m_{\text{normal}} = \frac{-1}{(-18)} = \frac{1}{18}$

Equation of line : $y - y_1 = m(x - x_1)$

gradient

coordinates of known point on line

→ known point is $P\left(\frac{1}{3}, 1\right)$

→ grad. nor $= \frac{1}{18}$

$\therefore \quad y - 1 = \frac{1}{18}\left(x - \frac{1}{3}\right)$

$54y - 54 = 3x - 1$

$3x - 54y + 53 = 0$

Q1

(Total 6 marks)

Turn over

2. The functions f and g are defined by

$$f(x) = \frac{5-x}{3x+2} \qquad x \in \mathbb{R}, x \neq -\frac{2}{3}$$

$$g(x) = 2x - 7 \qquad x \in \mathbb{R}$$

(a) Find the value of fg(5)

(2)

(b) Find f^{-1}

(3)

(c) Solve the equation

$$f\left(\frac{1}{a}\right) = g(a+3)$$

(4)

2. a) $f(g(5))$

$= f(2(5)-7) = f(3)$

$= \frac{5-3}{3(3)+2} = \frac{2}{11}$

b) to find $f^{-1}(x)$: $f(x) = \frac{5-x}{3x+2}$

① write the function using a "y" and set equal to "x" : $x = \frac{5-y}{3y+2}$

② rearrange to make y the subject : $3xy + 2x = 5 - y$

$y = \frac{5-2x}{3x+1}$

③ replace y with $f^{-1}(x)$:

$\therefore f^{-1}(x) = \frac{5-2x}{3x+1}$

domain :

$x \in \mathbb{R}$,

$x \neq -\frac{1}{3}$ } because denominator cannot = 0

↳ would be asymptote at $-\frac{1}{3}$

P 6 6 6 4 7 A 0 4 3 2

Question 2 continued

c) $f\left(\frac{1}{a}\right) = g(a+3)$

$$\frac{5-\left(\frac{1}{a}\right)}{3\left(\frac{1}{a}\right)+2} = 2(a+3)-7$$

$$\frac{5a-1}{3+2a} = 2a+6-7$$

$$5a-1 = (2a-1)(2a+3)$$

$$5a-1 = 4a^2+4a-3$$

$$4a^2-a-2=0$$

$$a = \frac{1 \pm \sqrt{33}}{8}$$

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Given that k is a positive constant,

(a) find

$$\int \frac{9x}{3x^2 + k}\, dx$$

(2)

Given also that

$$\int_2^5 \frac{9x}{3x^2 + k}\, dx = \ln 8$$

(b) find the value of k

(4)

3. a) $\int \frac{9x}{3x^2 + K}\, dx$

$u = 3x^2 + K$ $\quad \frac{du}{dx} = 6x \rightarrow dx = \frac{du}{6x}$

$\int \frac{9x}{u}\, dx = \int \frac{9\cancel{x}}{u} \times \frac{du}{6\cancel{x}}$

$= \frac{3}{2} \int \frac{1}{u}\, du$

$= \frac{3}{2} \ln|u| + c$

$= \frac{3}{2} \ln(3x^2 + K) + c$

P 6 6 6 4 7 A 0 8 3 2

Question 3 continued

b) $\displaystyle\int_2^5 \frac{9x}{3x^2+k}\,dx$

$= \left[\frac{3}{2}\ln(3x^2+k)\right]_2^5$

$= \frac{3}{2}\ln\left(3(5)^2+k\right) - \frac{3}{2}\ln\left(3(2)^2+k\right)$

$= \frac{3}{2}\left(\ln(75+k) - \ln(12+k)\right)$

$= \frac{3}{2}\left(\ln\left(\frac{75+k}{12+k}\right)\right)$ ← $\log_a b - \log_a c = \log_a\left(\frac{b}{c}\right)$

$\frac{3}{2}\left(\ln\left(\frac{75+k}{12+k}\right)\right) = \ln(8)$

$\ln\left(\frac{75+k}{12+k}\right) = \frac{2}{3}\ln(8)$

$\ln\left(\frac{75+k}{12+k}\right) = \ln\left(8^{\frac{2}{3}}\right)$ ← $a\log_b(c) = \log_b(c^a)$

$\therefore \frac{75+k}{12+k} = 4$

$75+k = 48+4k \rightarrow k = 9$

(Total 6 marks)

Q3

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4.

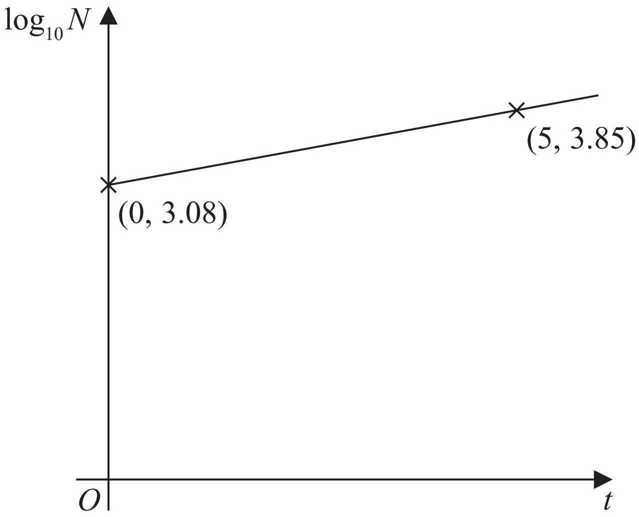


Figure 1

The number of subscribers to an online video streaming service, N, is modelled by the equation

$$N = ab^t$$

where a and b are constants and t is the number of years since monitoring began.

The line in Figure 1 shows the linear relationship between t and $\log_{10} N$

The line passes through the points (0, 3.08) and (5, 3.85)

Using this information,

(a) find an equation for this line.

(2)

(b) Find the value of a and the value of b, giving your answers to 3 significant figures.

(3)

When $t = T$ the number of subscribers is 500 000

According to the model,

(c) find the value of T

(2)

4.a) known points : (0, 3.08) & (5, 3.85)

$\underset{\text{(grad.)}}{m} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3.85 - 3.08}{5 - 0} = 0.154$

Question 4 continued

Equation of line : $y - y_1 = m(x - x_1)$ coordinates of known point on line

gradient

$$y - 3.08 = 0.154(t - 0)$$

$$y = 0.154t + 3.08$$

b) $N = a \times b^t$

$$\log_{10}(N) = \log_{10}(a \times b^t)$$

LOG RULES →

$$a\log_b(c) = \log_b(c^a)$$

$$\log_a b + \log_a c = \log_a(bc)$$

$$\log_a b = c \rightarrow a^c = b$$

$$\log_{10}(N) = \log_{10}(a) + \log_{10}(b^t)$$

$$\log_{10}(N) = \log_{10}(a) + t\log_{10}(b)$$

$$y = c + t\ m$$

COMPARE TO EQUATION FROM (a)

$$y = 3.08 + t(0.154)$$

$\log_{10}(a) = 3.08$ $\log_{10}(b) = 0.154$

$a = 10^{3.08} = 1200$ $b = 10^{0.154} = 1.43$

b) when $t = T$

$$N = 1200 \times 1.43^{t}$$

$$500\,000 = 1200 \times 1.43^{T}$$

$$1.43^{T} = \frac{1250}{3}$$

$$T = \log_{1.43}\left(\frac{1250}{3}\right)$$

$$= 17$$

5.

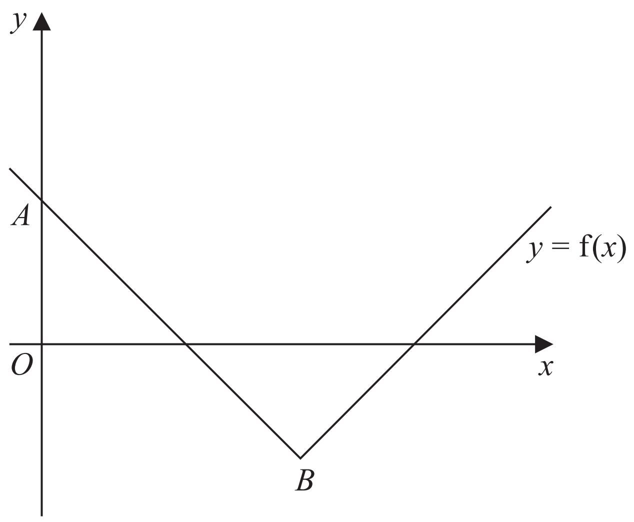


Figure 2

Figure 2 shows part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 9| - 2 \qquad x \in \mathbb{R}$$

and k is a positive constant.

The graph intersects the y-axis at the point A and has a minimum point at B as shown.

(a) (i) Find the y coordinate of A

(ii) Find, in terms of k, the x coordinate of B

(2)

(b) Find, in terms of k, the range of values of x that satisfy the inequality

$$|kx - 9| - 2 < 0$$

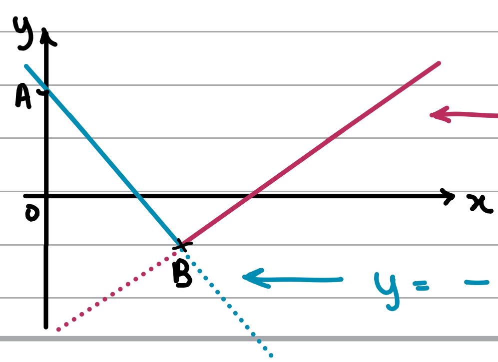
(3)

Given that the line $y = 3 - 2x$ intersects the graph $y = f(x)$ at two distinct points,

(c) find the range of possible values of k

(3)

5. a) (i) $f(x) = 2|2x-5| + 3 \qquad x \geq 0$



$\leftarrow y = (kx-9) - 2 = kx - 11$

$\leftarrow y = -(kx-9) - 2 = -kx + 7$

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Question 5 continued

A is y intercept of $y = -kx + 7$

$\therefore \quad A = 7$

(ii) B is the point at which the 2 lines meet

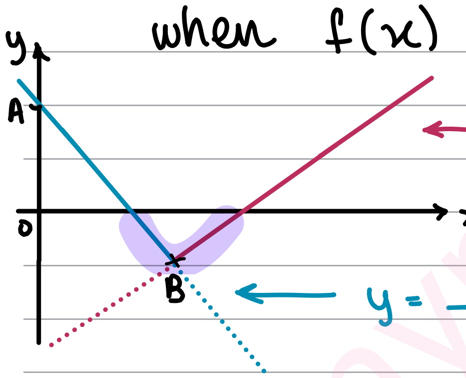
$$kx - 11 = -kx + 7$$

$$2kx = 18$$

$$x = \frac{9}{k}$$

b) $|kx - 9| - 2 < 0$

when f(x) is below the x axis



$\therefore$ between x-axis intercepts of both parts of f(x)

$-kx + 7 = 0 \qquad\qquad kx - 11 = 0$

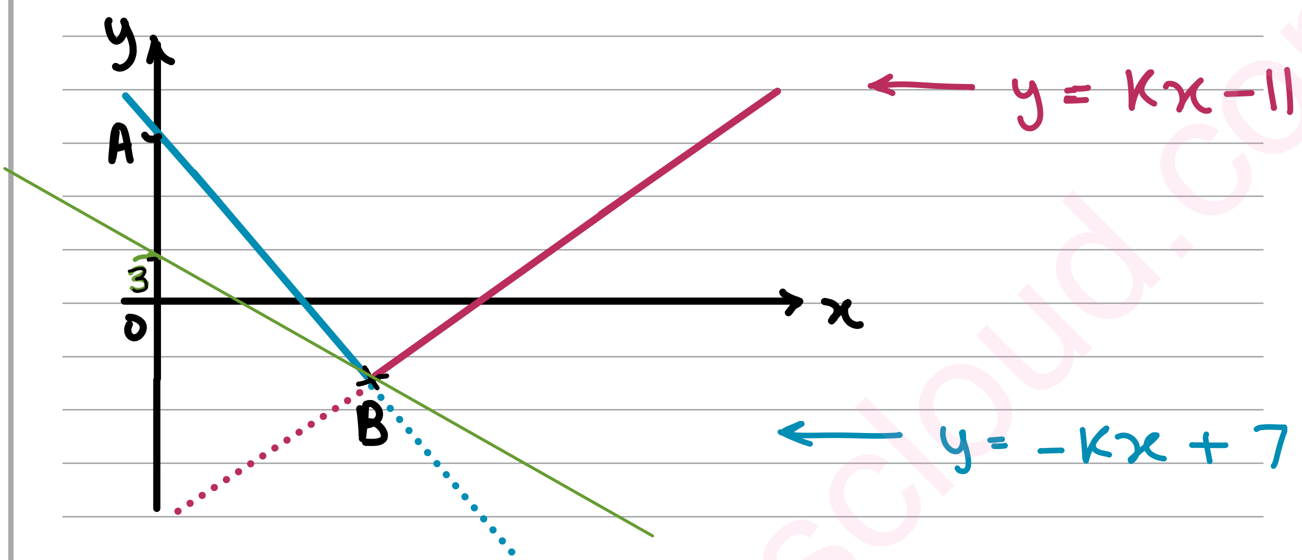
$x = \frac{7}{k} \qquad\qquad x = \frac{11}{k}$

$$\therefore \quad \frac{7}{k} < x < \frac{11}{k}$$

Question 5 continued

c) If the line $y = 3 - 2x$ intersects with $f(x)$ twice

└ it must intersect with both parts of $f(x)$



For $y = 3 - 2x$ to pass through $B \rightarrow \left(\frac{9}{K}, -2\right)$

$$-2 = 3 - 2\left(\frac{9}{K}\right) \rightarrow \frac{9}{K} = \frac{5}{2}$$

$$K = 3.6$$

* If $K < 3.6$ → $f(x)$ will become less steep as the gradient is $K / -K$

then there will not be 2 intersections

as B will be above the line $y = 3 - 2x$

* If $k = 3.6$, $y = 3 - 2x$ intersects with $f(x)$ once at B

$\therefore$ not 2 distinct solutions

$\therefore$ for 2 solutions → $K > 3.6$

6.

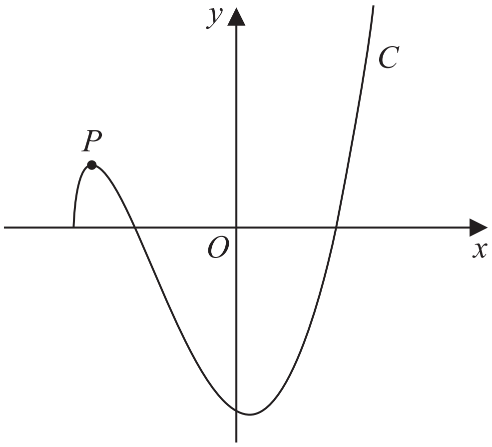


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The function f is defined by

$$f(x) = 5(x^2 - 2)(4x + 9)^{\frac{1}{2}} \qquad x \geqslant -\frac{9}{4}$$

(a) Show that

$$f'(x) = \frac{k(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$$

where k is an integer to be found.

(4)

(b) Hence, find the values of x for which $f'(x) = 0$

(1)

Figure 3 shows a sketch of the curve C with equation $y = f(x)$.

The curve has a local maximum at the point P

(c) Find the exact coordinates of P

(2)

The function g is defined by

$$g(x) = 2f(x) + 4 \qquad -\frac{9}{4} \leqslant x \leqslant 0$$

(d) Find the range of g

(3)

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Question 6 continued

6.a) $f(x) = 5(x^2-2)(4x+9)^{\frac{1}{2}}$ $x \geqslant -\frac{9}{4}$

$\boxed{\text{PRODUCT RULE : } y = uv \quad y' = u'v + uv'}$

$u = 5x^2 - 10$ $\frac{du}{dx} = 10x$

$v = (4x+9)^{\frac{1}{2}}$ $\frac{dv}{dx} = \frac{1}{2} \times 4 \times (4x+9)^{-\frac{1}{2}}$

$= 2(4x+9)^{-\frac{1}{2}}$

$f'(x) = (10x)\left((4x+9)^{\frac{1}{2}}\right) + (5x^2-10)\left(2(4x+9)^{-\frac{1}{2}}\right)$

$= \frac{10x(4x+9) + 10x^2 - 20}{(4x+9)^{\frac{1}{2}}}$

$= \frac{40x^2 + 90x + 10x^2 - 20}{(4x+9)^{\frac{1}{2}}}$

$= \frac{50x^2 + 90x - 20}{(4x+9)^{\frac{1}{2}}} = \frac{10(5x^2+9x-2)}{(4x+9)^{\frac{1}{2}}}$

$\therefore k = 10$

b) $f'(x) = 0$

$\frac{10(5x^2+9x-2)}{(4x+9)^{\frac{1}{2}}} = 0$

$5x^2 + 9x - 2 = 0$

$(5x-1)(x+2) = 0$ $x = \frac{1}{5} \cup -2$

Question 6 continued

c) From figure 3, we can see the 2 stationary points at which $f'(x) = 0$

P → where a local max occurs → x is negative

local minimum → x is positive

we had 2 values of x for which $f'(x) = 0$

$\therefore$ x coordinate of P is negative value = -2

$\therefore P = (-2, 5(2)(1)^{\frac{1}{2}})$

$= (-2, 10)$

d) $g(x) = 2f(x) + 4$ $\quad -\frac{9}{4} \leq x < 0$

between given domain, we can see from the graph:

max value of f(x) is at P → $f(x) = 10$

min value of f(x) is at the end of the domain → $f(0) = -30$

$\therefore f(-2) \leq f(x) < f(0)$

$10 \geq f(x) > -30$

$24 \geq 2(f(x)) + 4 > -56$

$\therefore -56 < g(x) \leq 24$

Leave blank

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\sin\theta\,(3\cot^2 2\theta - 7) = 13\sec\theta$$

can be written as

$$3\operatorname{cosec}^2 2\theta - 13\operatorname{cosec} 2\theta - 10 = 0$$

(4)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$, the equation

$$2\sin\theta\,(3\cot^2 2\theta - 7) = 13\sec\theta$$

giving your answers to 3 significant figures.

(4)

7. a) $2\sin(\theta)\left(3\cot^2(2\theta) - 7\right) = 13\sec(\theta)$

$$2\sin(\theta)\left(\frac{3\cos^2(2\theta)}{\sin^2(2\theta)} - 7\right) = \frac{13}{\cos(\theta)}$$

$$2\sin(\theta)\cos(\theta)\left(\frac{3\cos^2(2\theta) - 7\sin^2(2\theta)}{\sin^2(2\theta)}\right) = 13$$

DOUBLE ANGLE FORMULAE $\sin(2A) = 2\sin(A)\cos(A)$

$$\cancel{\sin(2\theta)}\left(\frac{3\cos^2(2\theta) - 7\sin^2(2\theta)}{\sin^{\cancel{2}}(2\theta)}\right) = 13$$

$$3\cos^2(2\theta) - 7\sin^2(2\theta) = 13\sin(2\theta)$$

$$3(1 - \sin^2(2\theta)) - 7\sin^2(2\theta) = 13\sin(2\theta)$$

$$3 - 10\sin^2(2\theta) = 13\sin(2\theta)$$

← divide both sides by $\sin^2(2\theta)$

$$3\operatorname{cosec}^2(2\theta) - 10 = 13\operatorname{cosec}(2\theta)$$

Question 7 continued

$$3\operatorname{cosec}^2(2\theta) - 13\operatorname{cosec}(2\theta) - 10 = 0$$

b) $0 < \theta < \frac{\pi}{2}$

let $\operatorname{cosec}(2\theta) = u$

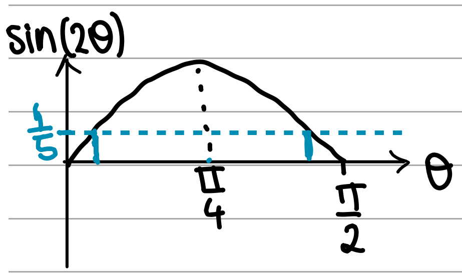
$$3u^2 - 13u - 10 = 0$$

$$(3u+2)(u-5) = 0$$

$$u = \operatorname{cosec}(2\theta) = -\frac{2}{3} \cup 5$$

$$\therefore \sin(2\theta) = \cancel{-\frac{3}{2}} \cup \frac{1}{5}$$

reject as $-1 \leq \sin(2\theta) \leq 1$



$$2\theta = 0.2014 \cup 2.94$$

$$\theta = 0.101 \cup 1.47$$

8.

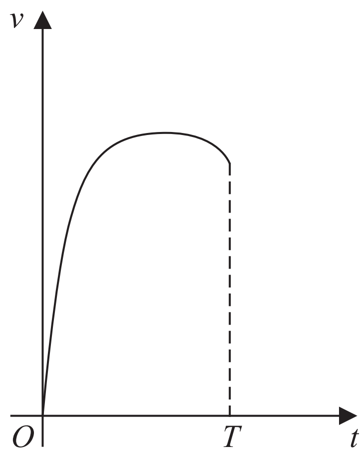


Figure 4

Figure 4 is a graph showing the velocity of a sprinter during a 100 m race.

The sprinter's velocity during the race, $v\,\text{m s}^{-1}$, is modelled by the equation

$$v = 12 - e^{t-10} - 12e^{-0.75t} \qquad t \geqslant 0$$

where t seconds is the time after the sprinter begins to run.

According to the model,

(a) find, using calculus, the sprinter's maximum velocity during the race.

(5)

Given that the sprinter runs 100 m in T seconds, such that

$$\int_0^T v \, dt = 100$$

(b) show that T is a solution of the equation

$$T = \frac{1}{12}\left(116 - 16e^{-0.75T} + e^{T-10} - e^{-10}\right)$$

(4)

The iteration formula

$$T_{n+1} = \frac{1}{12}\left(116 - 16e^{-0.75T_n} + e^{T_n-10} - e^{-10}\right)$$

is used to find an approximate value for T

Using this iteration formula with $T_1 = 10$

(c) find, to 4 decimal places,

(i) the value of T_2

(ii) the time taken by the sprinter to run the race, according to the model.

(3)

Question 8 continued

8. a) $v = 12 - e^{t-10} - 12e^{-0.75t}$ $\quad t \geqslant 0$

max speed occurs when $\frac{dv}{dt} = 0$

$$\boxed{\begin{array}{ll} y = e^{ax+b} \rightarrow & \frac{dy}{dx} = a \times e^{ax+b} \\ y = e^{kx} & \frac{dy}{dx} = ke^{kx} \end{array}}$$

$$\frac{dv}{dt} = -1(1)e^{t-10} - (12)(-0.75)e^{-0.75t}$$

$$= -e^{t-10} + 9e^{-0.75t}$$

If max speed when $t = T$

$$-e^{T-10} + 9e^{-0.75T} = 0$$

$$9e^{-0.75T} = e^{T-10}$$

$$e^{T-10+0.75T} = 9$$

$$1.75T - 10 = \ln(9)$$

$$T = 6.97 \text{ s}$$

$\therefore$ max $v = v(T)$

$$= 12 - e^{T-10} - 12e^{-0.75T} = 11.9 \text{ ms}^{-1}$$

Question 8 continued

b) $\int_0^T v \, dt$

$= \int_0^T 12 - e^{t-10} - 12e^{-0.75t} \, dt$

$= \left[12t - e^{t-10} \quad \frac{-12e^{-0.75t}}{-0.75} \right]_0^T$

$= 12(T) - e^{T-10} + 16e^{-0.75T} + e^{-10} - 16$

$= 12T - e^{T-10} + 16e^{-0.75T} + e^{-10} - 16$

if $\int_0^T v \, dt = 100$

$12T - e^{T-10} + 16e^{-0.75T} + e^{-10} - 16 = 100$

$T = \frac{1}{12}\left(116 - 16e^{-0.75T} + e^{T-10} - e^{-10}\right)$

c) (i) $T_{n+1} = \frac{1}{12}\left(116 - 16e^{-0.75T_n} + e^{T_n - 10} - e^{-10}\right)$

$T_1 = 10$

$T_2 = T_{1+1} = \frac{1}{12}\left(116 - 16e^{-0.75(10)} + e^{10-10} - e^{-10}\right)$

$= 9.7493$

(ii) $T_3 = 9.7306$ $\quad T_5 = 9.7293$

$T_4 = 9.7294$ $\quad T_6 = 9.7293$

consistent to 4 d.p.

$\therefore T = 9.7293$ (to 4 d.p.)

9.

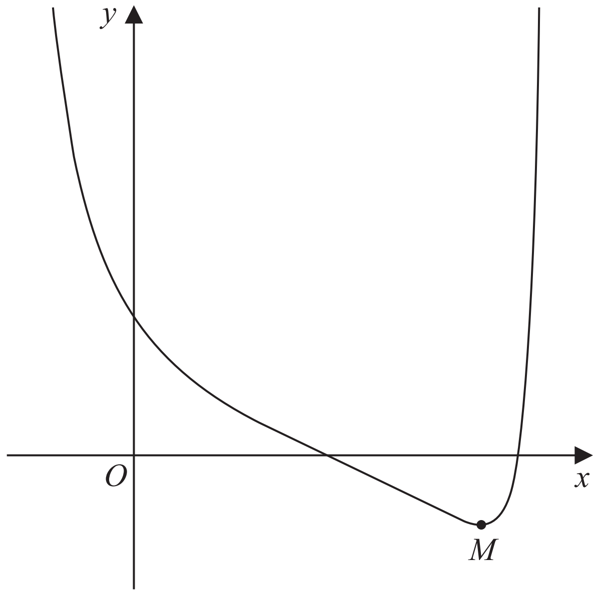


Figure 5

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 5 shows the curve with equation

$$y = \frac{1 + 2\cos x}{1 + \sin x} \qquad -\frac{\pi}{2} < x < \frac{3\pi}{2}$$

The point M, shown in Figure 5, is the minimum point on the curve.

(a) Show that the x coordinate of M is a solution of the equation

$$2\sin x + \cos x = -2$$

(4)

(b) Hence find, to 3 significant figures, the x coordinate of M.

(5)

9. a) If M is a minimum, $\frac{dy}{dx}$ at M = 0

$y = \frac{1 + 2\cos(x)}{1 + \sin(x)}$

Quotient rule for differentiating : $y = \frac{u}{v} \rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Question 9 continued

$u = 1 + 2\cos(x) \qquad \frac{du}{dx} = -2\sin(x)$

$v = 1 + \sin(x) \qquad \frac{dv}{dx} = \cos(x)$

$$\frac{dy}{dx} = \frac{(1+\sin(x))(-2\sin(x)) - (1+2\cos(x))(\cos(x))}{(1+\sin(x))^2}$$

$$= \frac{-2\sin(x) - 2\sin^2(x) - \cos(x) - 2\cos^2(x)}{(1+\sin(x))^2}$$

when $\frac{dy}{dx} = 0$

$$-2\sin(x) - 2\sin^2(x) - \cos(x) - 2\cos^2(x) = 0$$

$\sin^2(A) + \cos^2(A) = 1$

$\therefore \quad 2\sin(x) + \cos(x) + 2(\sin^2(x) + \cos^2(x)) = 0$

$2\sin(x) + \cos(x) = -2$

b) let : $2\sin(x) + \cos(x) = R\sin(x+\alpha)$

using compound angle formulae

$\sin(A+B)$

$= \sin(A)\cos(B) + \sin(B)\cos(A)$

$R(\sin x\cos\alpha + \sin\alpha\cos x) = 2\sin(x) + \cos(x)$

Compare expanded expressions

P 6 6 6 4 7 A 0 3 1 3 2

Question 9 continued

$R\sin x\cos\alpha + R\sin\alpha\cos x = 2\sin(x) + \cos(x)$

$R\cos\alpha = 2$ $\quad$ $R\sin\alpha = 1$

$\frac{\cancel{R}\sin\alpha}{\cancel{R}\cos\alpha} = \tan\alpha = \frac{1}{2}$

$\alpha = 0.464$

$(R\cos\alpha)^2 + (R\sin\alpha)^2$ $\quad$ $\cos^2 A + \sin^2 A = 1$ identity

$= R^2(\cos^2\alpha + \sin^2\alpha) = R^2(1)$

$= 2^2 + 1^2$

$= 5 \quad \therefore R^2 = 5$

$R = \sqrt{5}$

$2\sin(x) + \cos(x) = -2$

$\therefore \quad \sqrt{5}\sin(x + 0.464) = -2$

$\sin(x + 0.464) = \frac{-2}{\sqrt{5}}$

$x + 0.464 = -1.11 \cup 4.249 \cup \ldots$

however x is positive

$\therefore \quad x = 4.249 - 0.464$

$= 3.78$

Q9

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END

P 6 6 6 4 7 A 0 3 2 3 2